(1) $$y_t = \beta_1 + \beta_2 x_{2t} + \cdots + \beta_k x_{tk} + u_t$$

$$u_t \text{ iid } N(0, \sigma^2)$$

DGP - for ANY dataset this is the process that actually generated the data.

- we usually assume that it is much simpler than reality. I.e. the fully spec. model.

model - set of DGP.

LRM consists of ALL DGP that could generate (1) [Different $\beta_i \in R^k$ and $\sigma^2 > 0$]

CNLRM

CLRM

$$y_t = \beta_1 + \beta_2 x_{2t} + \cdots + \beta_k x_{kt} + u_t$$

u_t iid $(0, \sigma^2)$

u_t can have any dist

as long as $E(u_t) = 0$
$Var(u_t) = \sigma^2$

Model is misspecified when:

The DGP that generates the DATA does not belong to the model (set) of possible DGP under consideration.

$E(u_t) \neq 0$ omit vars. etc.

$Var(u_t) \neq \sigma^2$

u_t not iid

Bad function

nonlinearity.

$$\underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{u} \qquad u \sim (\underset{\sim}{0}, \sigma^2 I_T)$$

$$\hat{\beta} = (X^T X)^{-1} X^T \underset{\sim}{y}$$

$$\hat{\beta} = (X^T X)^{-1} X^T (X\beta + u)$$
$$= \beta + (X^T X)^{-1} X^T u.$$

$$E(\hat{\beta}) = \beta + E(X^T X)^{-1} X^T u$$
$$= \beta \quad \text{only if} \quad E(\ ) = 0$$

True if $E(\underset{\sim}{u} | X) = 0$

<u>Exogeneity</u>

each u_t indep of all X.

Weak exog. or predeterminedness

$E(u_t | \underset{\sim}{x}_t) = 0$

$$y_t = \alpha_1 + \beta y_{t-1} + u_t$$

here $\underset{\sim}{x}_t = \{1 \quad y_{t-1}\}$

For strong exog.

$E(u|X) = 0.$ OLS unbiased only in this case

Not met ~~since~~

$$u_{t-1} \xrightarrow{\text{CORR}} y_{t-1}$$

If $E(u_t | \underset{\sim}{x}_t) = 0$ OLS ~~may be~~ Biased.

u_t can be endog of y_{t-1}

even though it is ~~not~~

~~endog of ... y_{t+1}~~

u_{t-1} is not endog of y_{t-1}

See p. 14

$$E(u \mid X) = 0$$

$$\Rightarrow \quad E(X^T u) = 0$$

$$E(u \mid X) = 0 \qquad E(h$$

$$E_{X_2}\left[E_{X_1}(X_1 \mid X_2)\right] = E(X_1)$$ L of I Exp.

$$E\left[X_1 h(X_2) \mid X_2\right] = h(X_2) \cdot E(X_1 \mid X_2)$$

$$E(X^T u \mid X) = X^T E(u \mid X) = 0$$

since $E(u \mid X) = 0$