Large Sample Tests

t & F tests are only exact under some fairly strenuous conditions.

If $E(\underset{\sim}{u}|X) \neq 0$ only $E(u_t|\chi_t) = 0$ or u not normally dist then these tests are not exact.

In this case the justification for using these tests relies on asymptotic theory.

$$\underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{u} \qquad \underset{\sim}{u} \sim (0, \sigma^2 I_n)$$

$$E(u_t|\chi_t) = 0$$
$$E(u_t^2|\chi_t) = \sigma^2$$

$$\operatorname*{plim}_{n\to\infty} \frac{1}{n}\left(\frac{X^T X}{n}\right) = S_{xx} \text{ finite \& nonsingular}$$

then $\lambda_1, \lambda_2, \lambda_3 \xrightarrow{d} \chi^2_J/J$

if $R\beta = r$ is true.

$$F_{J,n-k} \xrightarrow{d} \chi^2_J / J$$

$$t_{n-k} \xrightarrow{d} N(0,1)$$

note also $t^2_{n-k} = F_{1,n-k}$

LS

$$\sqrt{n}(\hat{\beta}-\beta) \xrightarrow{d} N\left(0, \sigma^2 \operatorname*{plim}_{n\to\infty}\left(\frac{X^TX}{n}\right)^{-1}\right)$$

in finite samples replace plim with

$$\frac{X^TX}{n}$$

$$\sqrt{n}(\hat{\beta}-\beta) \overset{a}{\sim} N\left(0, \sigma^2\left(\frac{X^TX}{n}\right)^{-1}\right)$$

divide by $\sqrt{n}$

$$(\hat{\beta}-\beta) \overset{a}{\sim} N(0, \sigma^2(X^TX)^{-1})$$

$$\hat{\beta} \overset{a}{\sim} N(\beta, \sigma^2(X^TX)^{-1})$$

which provides the basis for inference.

Similarly

$\lambda_1, \lambda_2, \lambda_3$

$$\lambda_1 = \frac{(R\hat{\beta} - r)^T \left[R (X^T X)^{-1} R^T\right]^{-1} (R\hat{\beta} - r) / J}{\hat{\sigma}^2}$$

$\overset{a}{\sim} F_{J, n-k}$ if H_0 True which, for large n,

$\cong \chi^2_J / J$

CI + 5.5, 5.6, 5.7,

Confidence Interval

$$\underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{u} \qquad u|X \sim N(0, \sigma^2 I_n)$$

~~Suppose we~~

to test the hypothesis

$$H_0: \beta_i = \beta_0$$

$$H_A: \beta_i \neq \beta_0$$

we used a t-test.

$$t = \frac{\hat{\beta}_i - \beta_0}{}$$

CI

The same principle can be used to obtain Conf Interval.

$(1-\alpha)\%$ CI contains the unknown param with desired Probability

$$Pr(a < \beta_2 < b) = (1-\alpha)$$

use the fact that

$$Pr\left(-t_{c,\alpha/2} < t < t_{c,\alpha/2}\right) = 1-\alpha$$

$$< \frac{\hat{\beta}_2-\beta_2}{SE} <$$

$$\hat{\beta}_2 \pm t_{c,\alpha/2} \cdot \hat{\sigma}^2 (X^T X)^{-1}_{ii}$$

P-values

P-values can be used to test a hypothesis. It saves the step of having to look critical values up in a table. Use with caution in 1-sided t-test.

P-value measures the size of the rejection region for the stat you have computed assuming Ho: True.

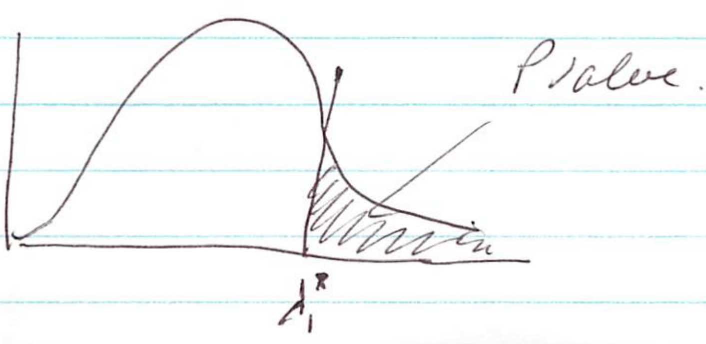
Examples.

$H_0: R\beta = r$

$H_A: R\beta \neq r$

$F = \lambda_1 \overset{a}{\sim} F_{J,n-k}$ if Ho True.

compute $\lambda_1 = \lambda_1^*$

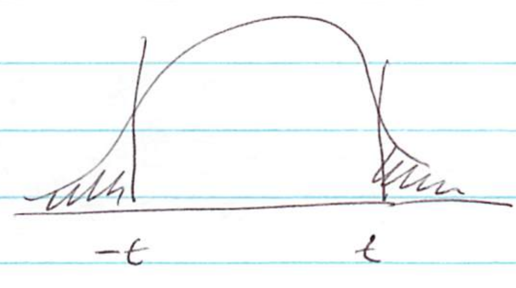


If Pvalue > α desired, then you cannot reject Ho: Reject only if Pvalue < α.

Most programs give you P-values. Just make sure the P-value given is for the test you want to perform

$H_0: \beta_2 = 0$
$H_A: \beta_2 \neq 0$

$$t = \frac{\hat{\beta}_2}{\hat{SE}(\hat{\beta}_2)} \overset{a}{\sim} t_{n-k}$$



Pvalue add the area in left & Right tails. These are printed in your Basic STATA output.

Heteroscedasticity - Consistent Cov. Matrices.

To use t-tests, F-tests, and C.I. that we've talked about you need a estimator of the CS Covariance Matrix.

$$y = X\beta + u \qquad u|X \sim (0, \sigma^2 I_n)$$

$$\hat{\beta} = (X^T X)^{-1} X^T y \qquad \hat{\beta} | X \sim N(\beta, \sigma^2 (X^T X)^{-1})$$

But, suppose

$$y = X\beta + u \qquad u|X \sim (0, \sigma^2 \Omega)$$

$$\Omega \neq I_n$$

This can occur when the errors do NOT have equal variance.

- heteroscedasticity

We can use LS to estimate β when $\underset{\sim}{u} | X \sim (0, \Omega)$

$$E(\hat{\beta}|X) = E(X^T X)^{-1} X^T (X\beta + u)$$
$$= \beta + E(X^T X)^{-1} X^T u = \beta.$$

$$Cov(\hat{\beta}) = E[(\hat{\beta} - E(\hat{\beta}))(\hat{\beta} - E(\hat{\beta}))^T] | X$$
$$= E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T] | X$$
$$= E[(X^T X)^{-1} X^T u u^T X (X^T X)^{-1}] | X$$
$$= E[(X^T X)^{-1} X^T \Omega X (X^T X)^{-1}]$$

This is an example of a SANDWICH Cov. Matrix.

$X^T \Omega X$ is sand.

between $(X^T X)^{-1}$

SAND. Cov. are almost never efficient.

White proposed the following consistent estimator of $X^T \Omega X$

$$X^T \begin{bmatrix} \hat{u}_1^2 & 0 & & 0 \\ 0 & \hat{u}_2^2 & & \\ & & \ddots & \\ 0 & & & \hat{u}_n^2 \end{bmatrix} X \equiv \widehat{X^T \Omega X}$$

and $\widehat{Var(\hat{\beta})} = (X^T X)^{-1} \widehat{X^T \Omega X} (X^T X)^{-1}$

and is referred to as heteroscedasticity-consistent cov. estimator or HCCME

STATA

regress y x2 x3 . . . xk, vce(robust)

gretl:

```
ols y const x2 x3 ... xk --robust
```

```
  ___  ____  ____  ____  ____tm
 /__    /   ____/   /   ____/
___/   /   /___/   /   /___/
  Statistics/Data Analysis
```

User: Testing exclusion restrictions {space -10}
Project: Fall 2004

```
      log:  C:\Temp\public_html\class\4213\f2004\mlb2.smcl
 log type:  smcl
opened on:   2 Nov 2004, 15:12:49
```

1 . regress lsalary years gamesyr bavg hrunsyr rbisyr, robust hc3

Regression with robust standard errors

Number of obs	=	353
F(5, 347)	=	129.16
Prob > F	=	0.0000
R-squared	=	0.6278
Root MSE	=	.72658

lsalary	Coef.	Robust HC3 Std. Err.	t	P>|t|	[95% Conf.	Interval]
years	.0688626	.0160111	4.30	0.000	.0373717	.1003536
gamesyr	.0125521	.0026933	4.66	0.000	.0072549	.0178493
bavg	.0009786	.0009684	1.01	0.313	-.0009262	.0028834
hrunsyr	.0144295	.0169739	0.85	0.396	-.0189552	.0478142
rbisyr	.0107657	.0073333	1.47	0.143	-.0036577	.0251891
_cons	11.19242	.2620281	42.71	0.000	10.67706	11.70778

2 . test (bavg) (hrunsyr)(rbisyr)

```
 ( 1)  bavg = 0
 ( 2)  hrunsyr = 0
 ( 3)  rbisyr = 0

       F(  3,   347) =    9.33
            Prob > F =    0.0000
```

3 . log close

```
      log:  C:\Temp\public_html\class\4213\f2004\mlb2.smcl
 log type:  smcl
closed on:   2 Nov 2004, 15:13:53
```

gretl

```
set hc_version 3
ols lsalary 0 years gamesyr bavg hrunsyr rbisyr --robust
restrict
 b[bavg]=0
 b[hrunsyr]=0
 b[rbisyr]=0
end restrict
```

Result:

Test statistic: Robust F(3, 347) = 9.33051, with p-value = 6.00713e-006

Alternative forms

use $\hat{u}_t^2 \left(\frac{n}{n-k}\right)$ - HC_1

$\hat{u}_t^2 / (1-h_t)$ HC_2

where h_t is $x_t^T (X^T X)^{-1} x_t^T$

t^{th} diag element of P_x or

the "hat" matrix.

$\hat{u}_t^2 / (1-h_t)^2$ HC_3

Stata

regress y x2 x3 . . . xk, robust HC3

```
gretl:
set hc_version 3
ols y const x --robust
```

When does het matter?

Basically, only when heteroscedasticity is related to squares & cross products of X's.

Delta Theorem

Spec. Tests

Reset - Functional Form.
gretl: reset

B-P - Heterosced. imtest, white
gretl: not fully implemented in 1.9.7. Do it manually.

White's - "
gretl: modtest --white

BP If installed $H_0: \sigma_i^2 = \sigma^2$

bpagan varlist $H_A: \sigma_i^2 = h(\alpha_1 + \alpha_2 z_2 + \dots)$

χ^2_J

$= \frac{1}{2} MSS$ $\alpha_1 + \alpha_2 z_2$

White's Imtest, white. $H_0: \sigma_i^2 = \sigma^2$

$H_A: \sigma_i^2 \neq \sigma^2$

$\hat{e}_i^2 = \beta_1 + \beta_2 x +$ 2 squares + cross prod $nR^2 \sim \chi^2_{J-1}$

het test varlist

$H_0: \sigma_i^2 = \sigma^2$

$H_1: \sigma_i^2 = \exp\{\alpha_1 + \alpha_2 z_2 + \dots + \alpha_s z_s\}$ χ^2_{J-1}