Generalized IV Estimator

Consider a set of instruments W where $W_{n \times \ell}$ and $\ell > k$. Here we have more instruments than needed.

- overidentified.

Basically, there is more than one way to specify the moment condition

$$\underline{E}\left[W^T(y - X\beta)\right] = 0$$

$$(\ell \times n)\left[(n \times 1)\ (n \times k)(k \times 1)\right] = \underset{\sim}{0}_{\ell \times 1}$$

i.e., there are ℓ equations and only k unknowns.

Asymptotic Cov Matrix

$$\sigma^2 \operatorname*{plim}_{n\to\infty}\left(\frac{1}{n}\, X^T W (W^T W)^{-1} W^T X\right)^{-1} \qquad \underset{\text{ETM}}{(8\text{-}17)}$$

optimal choice of instruments, W is the set that minimizes this.

Note: suppose $W = X$ and $E[u/X] = 0$

$$\sigma^2 \operatorname*{plim}_{n\to\infty}\left(\frac{1}{n}\, X^T X (X^T X)^{-1} X^T X\right)^{-1}$$

$$= \sigma^2 \operatorname*{plim}_{n\to\infty}\left(\frac{X^T X}{n}\right)^{-1} =$$

Asy. Cov of LS which is BLUE

X is optimal!

When $\ell = k \equiv$ Just Identified

$\ell < k \equiv$ underidentified
not identified or.

~~####~~ Solutions

(1) Drop instruments

This discards information
which is almost never a
good idea, especially if
instruments are <u>STRONG</u>

(2) Take a linear combination
of instruments

Take k linear combination of ℓ
columns

$$\underset{n\times\ell}{W}\,\underset{\ell\times k}{J} \quad \text{where } J\ \ell\times k$$

$$n \times k$$

The trick is to find the optimal J!

<u>Optimal</u> J

Choose J that minimizes the Asymptotic Covariance Matrix of the IV Estimator.

$$(WJ)^T(y - X\beta) = 0$$

Let $WJ = W(W^TW)^{-1}W^TX$

So, $J = (W^TW)^{-1}W^TX$

which is LS applied to

$$X = W\tilde{J} + U$$

So, $WJ = \hat{X} = W(W^TW)^{-1}W^TX$

from this Regression.

$$WJ = W(W^TW)^{-1}W^TX = P_W X = \hat{X}$$

$$=$$

Then

$$(WJ)^T(y - X\beta) = 0$$

$$X^T P_W (y - X\beta) = 0$$

$$X^T P_W y - X^T P_W X \hat{\beta}_{IV} = 0$$

$$\hat{\beta}_{IV} = (X^T P_W X)^{-1} X^T P_W y$$

$$= \left(X^T W (W^TW)^{-1} W^T X\right)^{-1} X^T W (W^TW)^{-1} W^T y$$

SAme As GLS estimator.

Also,

$$\hat{X}^T(y - X\beta) = 0$$

$\hat{X}$ Maximizes "Fit" Between X & W (Because its least squares). For Instruments W, This Choice of J is <u>optimal</u>.

2SLS

$$\hat{\beta}_{IV} = (X^T P_w X)^{-1} X^T P_w \underset{\sim}{y}$$

$$= (X^T P_w^T P_w X)^{-1} X^T P_w^T \underset{\sim}{y}$$

$$P_w X = W (W^T W)^{-1} W^T X = \hat{X}$$

$$\hat{\beta}_{IV} = (\hat{X}^T \hat{X})^{-1} \hat{X}^T \underset{\sim}{y}$$

or is the result of

A regression

$$y = \hat{X} \beta + \underset{\sim}{u}$$

This means that you can get the same result by running 2 regressions

(1) $X = W\check{J} + V$

stage 1

$$J = (W^T W)^{-1} W^T X$$

$$\hat{X} = WJ$$

(2) $y = \hat{X}\beta + u$

stage 2 $\hat{\beta}_{IV} = (\hat{X}^T \hat{X})^{-1} \hat{X}^T y$

Hence the Name:
Two Stage Least Squares

Warning

The correct covariance matrix is Estimated

$$\hat{\sigma}^2 (X^T P_W X)^{-1} = \hat{\sigma}^2 (\hat{X}^T \hat{X})^{-1}$$

$$\hat{\sigma}^2 (X^T W (W^T W)^{-1} W^T X)^{-1}$$

where $\hat{\sigma}^2 = \dfrac{(y - X\hat{\beta}_{IV})^T (y - X\hat{\beta}_{IV})}{n-k}$

NOTE: This is NOT The SAME AS what would Be Produced by a Regression Package if you use $\hat{X}$ in place of X.

wrong $\hat{\sigma}^2 = (y - \hat{X}\hat{\beta}_{IV})^T (y - \hat{X}\hat{\beta}_{IV}) / n-k$

correct $\hat{\sigma}^2 = (y - X\hat{\beta}_{IV})^T (y - X\hat{\beta}_{IV}) / n-k.$

So, Use Proper IV SOFTWARE And don't do this in 2 Steps.