Discrete & Limited Dependent Variable Models

Although regression is useful, it is not suitable for some problems. It is probably not suitable ~~for~~ when the dependent variable is discrete or when it is continuous, but limited in range.

- Binary Response
- Discrete dependent variable (of which BR is a special case)
- Count DATA

Lim dep - some obs = 0
AUTO expenditures

Tobit

Trunc Regression

Sample selectivity
Duration models

Discrete Choice Models

Dependent variable takes integer values 0, 1, 2, ...

Sometimes the values are meaningful and sometimes NOT.

1. COUNT DATA

   Ex # patents $y = 0, 1, 2, 3 \ldots$

2. LFP Binary Choice.

$$y = \begin{cases} 1 & \text{if yes} \\ 0 & \text{NO.} \end{cases}$$

   Coding is done for convenience and 1 or 0 have no numerical meaning – Measures a quality.

3. Rankings

$$y = \begin{cases} 1 & \text{oppose} \\ 0 & \text{neutral} \\ -1 & \text{support} \end{cases}$$

Numbers are ordinal (NOT Cardinal)

you could just as easily code

$$y = \begin{cases} 1 & \text{oppose} \\ 2 & \text{neutral} \\ 3 & \text{support.} \end{cases}$$

4) Categorical choice

$$y = \begin{cases} 0 & \text{Bus} \\ 1 & \text{car} \\ 2 & \text{Bike} \\ 3 & \text{other.} \end{cases}$$

neither a ranking, nor a count.

5) Combinations of the above

can vary based on
characteristics of the choice
or the individual making the
choice.

Binary Response (BR)

Here, the dependent variable can take one only 1 of 2 values. — 0 and 1

Let P_t denote probability that $y_t = 1$ conditional on the information set Ω_t which consists of exogenous and predetermined vars.

$$P_t \equiv \Pr(y_t = 1 \mid \Omega_t) = E(y_t \mid \Omega_t) \quad *$$

⇒ that BR can be thought of as modeling a conditional expectation

suppose Ω_t can be represented by X_t' a $1 \times k$ vector of explanatory vars.

$$* \quad E(y_t \mid \Omega_t) = 0 \cdot \Pr(y_t = 0 \mid \Omega_t) + 1 \cdot \Pr(y_t = 1 \mid \Omega_t)$$

#

We could use a regression model for this

$$E(y_t \mid X_t'\beta) = X_t'\beta + e_t$$

But, such a model fails to impose

$$0 \le E(y_t \mid X_t'\beta) \le 1$$

There are a number of ways to impose this restriction. The ones we'll consider ensure

$$0 < P_t < 1$$

by specifying

$$P_t \equiv E(y_t | \Omega_t) = F(x_t' \beta)$$

$x_t' \beta$ is an index function which maps to a scalar and $F(\cdot)$ is a transformation function which has the properties

$$F(-\infty) = 0 \quad , \quad F(\infty) = 1 \quad \text{and}$$

$$f(x) \equiv \frac{d F(x)}{d x} > 0$$

These properties are actually those that define a CDF. So $F(x_t'\beta)$ is really just a CDF that depends on the value of the index

These properties also assure us that $F(\cdot)$ is nonlinear $\Rightarrow$ marginal effects are also not linear

$$\frac{\partial P_t}{\partial x_{ti}} = \frac{\partial F(x_t'\beta)}{\partial x_{ti}} = f(x_t'\beta)\beta_i$$

where $f(\,)$ is p.d.f and β_i is i^{th} element of $\underset{\sim}{\beta}$.

For the Logit, Probit models, $f(x_t'\beta)$ ~~almost~~ achieves a max at $x_t'\beta = 0$ and then falls as $|x_t'\beta|$ increases. $\Rightarrow$ largest marginal effects occur when index is close to zero and get smaller the further out in the tail you go.

Logit & Probit

These are models of Binary choice:

yes -

no -

$$y_i = \begin{cases} 1 & \text{choice } A \\ 0 & \text{choice } B. \end{cases}$$

Let $P_i = \Pr[y_i = 1]$

$(1-P_i) = \Pr[y_i = 0]$

There are several Economic Rationals which yield these statistical models:

Index Function: outcome of choice is a reflection of an underlying regression. Marg bene > MC, then yes. Otherwise no. $y^* = \beta' x + \varepsilon$ $\quad \varepsilon \sim N(0, \sigma^2 I)$

~~Random Utility~~ $\quad y^*_i = x_i'\beta + e_i$

y^* is unobservable.

$y_i = 1$ if $y^* > 0$

$y = 0$ if $y^* < 0$

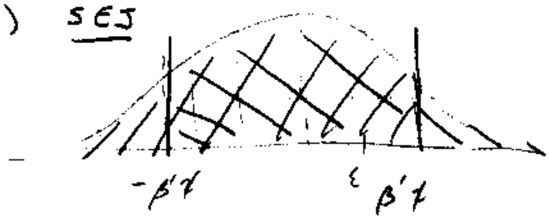
$\underset{k(1 \times k)(k \times 1)}{\beta' x}$ is the index function

$$\text{Prob}(y=1) = \Pr[y^* > 0] = \Pr[x_i'\beta + \varepsilon_i > 0] = \text{Prob}[\varepsilon_i > -x_i'\beta]$$

If the distribution of ε is symmetric then

$$Pr[y^* > 0] = Pr[\varepsilon < x_i'\beta] = F(x_i'\beta) = \int_{-\infty}^{\beta' x_i} f(x)\,dx$$

See Nakosteen & Zimmer (1980) SEJ



$Pr[\varepsilon > -x_i'\beta]$ \\\\\\\\

$Pr[\varepsilon < x_i'\beta]$ ///////

Random Utility Models

$$U_{i0} = E(U_{i0}) + e_{i0}$$
$$U_{i1} = E(U_{i1}) + e_{i1}$$

Utility of individual i associated with choice 0
" " " " " " 1

Utility - unobservable and a function of some systematic part and some random part.

We usually assume that the systematic part is linear in some set of observable vars.

$U_{i0} = E\bar{U}_{i0} + e_{i0} = \alpha_0 + z_{i0}'\delta + w_i'\gamma_0 + e_{i0}$

$U_{i1} = E U_{i1} + e_{i1} = \alpha_1 + z_{i1}'\delta + w_i'\gamma_1 + e_{i1}$

notice,

z_i are observable characteristics associated with the individual choice $\{0,1\} <$ car / Bus.

δ are not choice specific

↳ respond equally to time < car / Bus

w_i are individual specific, not choice specific and usually include info on age, income etc. Note, how $\gamma_0 \neq \gamma_1$ – some individuals characteristics may affect utility of each choice differently. \$100,000/year – diff < car / Bus.

If $y_i = 1$ – $U_{i1} > U_{i0}$

$$P(y_i=1) = P_i = Pr[U_{i1} > U_{i0}]$$
$$= Pr[(e_{i0} - e_{i1}) \leq (EU_{i1} - E_{i0})]$$
$$= Pr[(e_{i0} - e_{i1}) \leq (\alpha_1 - \alpha_0) + (z_{i1}' - z_{i0}')\delta + w_i'(\gamma_1 - \gamma_0)]$$

notice that $(z_{i1}' - z_{i0}')$ is difference not level in the characteristics. – no difference ⇒ no effect.

This leads to a situation where $e_i^* = (e_{i0} - e_{i1})$ we are looking for Prob that a R.V. is less than some const.

$$\Pr[y_i = 1] = P_i = \Pr\left[(e_i^*) \le x_i'\beta\right] = F(x_i'\beta)$$

where $x_i' = (1, (z_{i1} - z_{i0})', \underset{\sim}{w}_i')$

$$\underset{\sim}{\beta} = \begin{bmatrix} (\alpha_1 - \alpha_0) \\ \delta \\ \gamma_1 - \gamma_0 \end{bmatrix}$$

Probit — If $e_i^* \sim N(0, \sigma^2 I) \Rightarrow F(x_i'\beta) = \int_{-\infty}^{x_i'\beta} \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-t^2/2}\, dt$

Linear Prob — if $e_i^* \sim U(0,1) \Rightarrow F(x_i'\beta) = x_i'\beta$

if $e_i \sim$ logistic $\Rightarrow F(x_i'\beta) = \frac{1}{1 + e^{-x_i\beta}}$