(Greene p. 780)

Properties of Probit MLE

April 3

Probit MLE will be inconsistent

(1) if errors are heteroskedastic

(2) in presence of unmeasured heterogeneity, omitted variables (even if orthog to other regressors)

(3) nonlinearity of functional form (index function)

(4) if you ~~m~~ are incorrect about normality of errors. (There are a few exceptions

Linear Probability Model

$$E(y=1) = \beta_1 + \beta_2 x_2 + \cdots + \beta_k x_k$$

note that the β_i measure
<u>marginal effects</u>

$$\frac{\partial P_i}{\partial x_i} = \beta_i \qquad \neq f(x_{it}^T\beta)\beta_i$$

Unless the range of the xs is severely restricted, then this is probably NOT a very good discription of the population response. There would be feasible values of x that would take us outside of the $0 < x_i'\beta < 1$ interval.

* So, at best, this is viewed as a convenient approximation to the underlying model.

Recall, y is Bernoulli and as such

$$P_i = E(y|x) = \beta_1 + \beta_2 x_2 + \cdots + \beta_k x_k = x_i'\beta \quad (1)$$

$$P_i(1-P_i) = Var(y_i|x_i) = x_i'\beta(1 - x_i'\beta) \quad (2)$$

(1) $\Rightarrow$ OLS is unbiased, consistent for β!

(2) $\Rightarrow$ Heterosced. cov estimation yields valid tests!

⇒ The overall F-stat is actually valid.
(since if $\beta_2 = \cdots = \beta_K = 0$ then
$x_i'\beta(1 - x_i\beta)$ is constant!

⇒ FGLS is asympt. more efficient than
OLS.

$$\hat{\sigma}_i^2 = x_i'\hat{\beta}(1 - x_i'\hat{\beta})$$

of course, only useful if $\hat{\sigma}_i^2 > 0$!

⇒ Implies that unit increases in x_j
always change P_i by same
amount no matter what the
initial value of x_j; eventually this
will take us outside $0 < \hat{y}_i < 1$ interval.

Overall, NOT That Bad.

Probit

Specification Issues

Neglected Heterogeneity — suppose we've omitted some variable (unmeasurable or otherwise omitted variable) that is **independent** of $\underline{x}$

$$P(y=1 \mid \underline{x}, c) = F(x_i'\beta + \gamma c)$$

x_i' $1 \times K$, $x_1 = 1$

c is a scalar

γ. unknown.

Now, suppose c is independent of x and $c \sim N(0, \tau^2)$

(stronger than no covariance since the entire dist of c is indep of x.) $\Rightarrow f(x, c) = f(x) g(c)$

$\Rightarrow$ Latent variable form

$$y^* = x_i'\beta + \gamma c + e \qquad e \sim N(0,1)$$

$$\gamma c + e \sim N(0, 1 + \gamma^2 \tau^2)$$

$$\therefore P(y=1) = P(\gamma c + e > -x_i'\beta) = F(x_i'\beta / \sigma)$$

where $\sigma = \sqrt{1 + \gamma^2 \tau^2}$

Probit is consistent for β/σ

($\hat{\beta}$ is result of Probit of y on x)

$$\hat{\beta}_j \xrightarrow{p} (\beta_j/\sigma). \quad \text{But} \quad \sigma = \sqrt{\gamma^2\sigma_c^2+1} > 1$$

$$\therefore \left|\beta_j/\sigma\right| < |\beta_j|$$

This is sometimes called <u>ATTENUATION BIAS</u> in estimating β_j in the presence of neglected heterogeneity.

Is this a problem?

Well, yes & no. Probit is inconsistent for β_j, but it is consistent for β_j/σ. Still get right signs and significance.

Partial Effects

$$\frac{\partial P(y=1 \mid x, c)}{\partial x_j} = \beta_j f(x_i'\beta + \gamma c)$$

But, c is unobservable and γ <u>can't be estimated</u>.

Suppose $E(c)=0$ AND we were satisfied with evaluating this partial effect at $c=0$

Still have a problem

Probit estimates $\beta_j/\sigma \, f(x_i'\beta/\sigma)$

The presence of σ means more attenuation Bias.

β_j/σ is smaller but $f(x_i'\beta/\sigma)$ is Bigger.

Net effect is uncertain.

What you can do is get what is called the average Partial effect (APE)

For given x, you average the Partial effect across all values of c.

Let x^o be the desired value of x^o.

$$E_c\left[\beta_j f(x_i^{o\prime}\beta + \gamma c)\right] = \beta_j/\sigma \, f(x_i^{o}\beta/\sigma)$$

when c is Normal. Hence, Probit consistently estimates this APE.

Bottom line: Unless the actual size of β_j means something, omitted heterogeneity in Probit is no big deal. Ignoring it you get consistent est of APE.

If c is not independent of X then you have serious problems. Can't estimate APE consistently (just like in linear Regression).

Probit w/ Heteroskedasticity.

$$y_t^* = x_t'\beta + e_i$$

$$Var(e_i) = \left(e^{z_i'\gamma}\right)^2$$

z cannot include a constant!

$$\ell = \sum y_i \ln F\left(\frac{x_i'\beta}{\exp\{z_i'\gamma\}}\right) + (1-y_i)\ln\left[1 - F\left(\frac{(x_i'\beta)}{\exp\{z_i'\gamma\}}\right)\right]$$

$$\frac{\partial \ell}{\partial \beta} = \sum_1^T \left(\frac{f_i(y_i - F_i)}{F_i(1-F_i)}\right) \exp\{-z_i'\gamma\} x_i$$

$$\frac{\partial \ell}{\partial \gamma} = \sum_1^T \frac{f_i(y_i - F_i)}{F_i(\cancel{y_i} - F_i)} \exp\{-z_i'\gamma\} z_i(-x_i'\beta)$$

Knapp & Seaks 1992

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