Counts

- # Times an event occurs
- # patents
- visits

Could be zero for a SUBSTANTIAL portion of pop'l.

Responses are discrete, ordered. But
- outcomes have cardinal value
- No upper bound.

Poisson & Negative Binomial Models

$$y_i = \{0, 1, 2, \dots\}$$
$$x_i^T = \{1 \ x_{i2} \cdots x_{ik}\} \text{ set of charact.}$$

Assume $\underline{E}(y_i | x_i) = \exp\{x_i^T \underline{\beta}\}$ (1)

This relates the Avg outcome to the characteristics. To be able to derive Probabilities, we'll need A distrib.

Several Choices

(1) Poisson

(2) Neg Bin I

(3) Neg Bin II

Poisson

$$\Pr(y_i = y) = \frac{\exp\{-\lambda_i\}\,\lambda_i^{y}}{y!} \qquad y = 0, 1, 2, 3 \ldots$$

$$E(y_i) = \lambda_i$$
$$Var(y_i) = \lambda_i$$

equidispersion

Then, model $\lambda_i = \exp\{ x_i^T \beta \}$

Hence, $E(y_i \mid x_i) = \exp\{ x_i^T \beta \}$

$Var(y_i \mid x_i) = \exp\{ x_i^T \beta \}$

(2) $$\ell = \log L = \sum_{i=1}^{n} \left[-\lambda_i + y_i \log(\lambda_i) - \log y_i! \right]$$

$$= \sum_{i=1}^{n} \left(-\exp\{x_i^T \beta\} + y_i (x_i^T \beta) - \log y_i! \right)$$

For computational reasons, drop $\log y_i!$
(it does not depend on β anyway)

$$\frac{\partial \ell}{\partial \beta} = \sum_{i=1}^{n} \left(y_i x_i - \exp\{x_i^T \beta\} x_i \right)$$

$$= \sum_{i=1}^{n} \left(y_i - \exp\{x_i^T \beta\} \right) x_i = 0 \quad (3)$$

$$= \sum_{i=1}^{n} \varepsilon_i x_i = 0 \quad (3.a)$$

$$E(y_i \mid x_i) = \exp\{x_i^T \beta\}$$

$$\Rightarrow \quad y_i = \exp\{x_i^T \beta\} + \varepsilon_i$$

$$E(\varepsilon_i \mid x_i) = 0$$

(3) and (3a) are Sample Moment Conditions. Consequently. what ever maximizes (2) is consistent as long as (1) is true, (even if y_i is NOT Poisson distributed). An estimator that possesses this quality is ~~This~~ called

Quasi-Maximum likelihood.

An estimator based on the wrong likelihood is consistent if the moment conditions implied by the likelihood eqn's (f.o.c.) are satisfied.

The trick is to estimate the correct covariance matrix (sandwich)

$$\mathrm{Cov}(\hat{\beta}_{MLE}) = I(\beta)^{-1} = \left[E(\exp\{x_i^T\beta\}x_i x_i^T)\right]^{-1}$$

$$\mathrm{Cov}(\hat{\beta}_{QMLE}) = I(\beta)^{-1} J(\beta) I(\beta)^{-1}$$

$$J(\beta) = E\left[(y_i - \exp\{x_i^T\beta\})^2 x_i x_i^T\right]$$
$$= E\left[\varepsilon_i^2 x_i x_i^T\right]$$

Basically, this looks like White's sandwich we used in linear regression models.

Replace expectations with sample averages and coeffs, β, with their MLE (or QMLE).

Notes: Fine for estimating coeffs, But not useful for estimating Probabilities.

e.g. Pr(5 patents in year)
we could estimate the Avg
in a year.

Robust will allow over-dispersion in est of parameters, though.

$$\text{i.e.}\quad V(y_i \mid x_i) = E(\varepsilon_i^2 \mid x_i) > \exp(x_i^T\beta)$$

Neg Bin I

$$\mathrm{Var}(y_i \mid x_i) = (1+\delta^2)\exp\{x_i^T\beta\} \qquad (4)$$

only consistent if (4) valid. ~~QMLE~~
NO QMLE.

Neg Bin II

$$\mathrm{Var}(y_i \mid x_i) = \left(1+\alpha^2\exp\{x_i^T\beta\}\right)\exp\{x_i^T\beta\}$$

$$\alpha^2 > 0$$

(1) Poisson Model

- equi dispersion

(2) NegBIN I

- Variance exceeds the mean, but ratio same for all obs $(1+\delta^2)$

(3) Neg Bin II

- Variance increases as the cond mean increases

Marginal Effects

$$\frac{\partial E(y_i|x_i)}{\partial x_{ik}} = \exp\{x_i^T\beta\}\beta_k$$

same sign.

Semi elasticity

$$= \frac{\partial E(y_i|x_i)}{\partial x_{ik}} \cdot \frac{1}{E(y_i|x_i)} = \beta_k$$

Relative change in the conditional mean if the k^{th} regressor changes by 1 unit (c.p.)

For Binary Regressor

$$\frac{E\{y_i \mid x_{ik}=1, x_i'\}}{E\{y_i \mid x_{ik}=0, x_i'\}} = \exp\{\beta_k\}$$

Conditional mean is $\exp\{\beta_k\}$ larger if $x_{ik}=1$ instead of 0.

See Verbeek