Bivariate Probit

$$y_1^* = x_1^T \beta_1 + \varepsilon_1 \qquad y_1 = 1 \text{ if } y_1^* > 0, \ 0 \text{ o/w.}$$
$$y_2^* = x_2^T \beta_2 + \varepsilon_2 \qquad y_2 = 1 \text{ if } y_2^* > 0, \ 0 \text{ o/w.}$$

$$\left(\begin{matrix} \varepsilon_1 \\ \varepsilon_2 \end{matrix} \,\middle|\, x_1, x_2 \right) \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{matrix} 1 & \rho \\ \rho & 1 \end{matrix} \right)$$

This model can be used in a couple of other situations.

e.g. LAZARTE ALCALA, 2011

Endogenous Regressor, Recursive

$$w^* = x_1^T \beta_1 + \varepsilon_1 \qquad w = 1 \text{ if } w^* > 0, \ 0 \text{ o/w.}$$
$$y^* = x_2^T \beta_2 + \gamma w + \varepsilon_2 \quad y = 1 \text{ if } y^* > 0, \ 0 \text{ o/w}$$

Endogenous Sampling

$$s^* = x_1^T \beta_1 + \varepsilon_1 \qquad s = 1 \text{ if } s^* > 0, \text{ o/w.}$$
$$y^* = x_2^T \beta_2 + \varepsilon_2 \qquad y = 1 \text{ if } y^* > 0, \ 0 \text{ o/w.}$$

y, x_2 observed only when $s = 1$.

Bivariate Normal CDF

$$\text{Prob}(X_1 < x_1,\ X_2 < x_2) = \int_{-\infty}^{x_2}\int_{-\infty}^{x_1} \phi_2(z_1, z_2, \rho)\, dz_1\, dz_2$$

$$\phi_2(x_1, x_2, \rho) = \frac{e^{-1/2(x_1^2 + x_2^2 - 2x_1x_2)/1-\rho^2}}{2\pi(1-\rho^2)^{1/2}}$$

let $q_{i1} = 2y_{i1} - 1$ $\quad (-1, 1)$

$q_{i2} = 2y_{i2} - 1$

$z_{ij} = x_{ij}^T \beta_j$ and $w_{ij} = q_{ij} z_{ij}$

and $\rho_i^* = q_{i1} q_{i2} \rho$

The log-likelihood

$$\ln L = \sum_{i=1}^{n} \ln P_i)$$

$$P_i = \text{Prob}(Y_1 = y_{i1}, Y_2 = y_{i2} \mid x_1, x_2)$$

$$= \Phi_2(w_{i1}, w_{i2}, \rho_i^*)$$

Φ_2 is Bivariate Normal CDF

ϕ_2 is " " p.d.f.

The likelihood Equations

$$\frac{\partial \ln L}{\partial \beta_j} = \sum_{i=1}^{n} \left(\frac{q_{ij}\, g_{ij}}{\Phi_2} \right) \underset{\sim}{x}_{ij} \quad j=1,2$$

$$\frac{\partial \ln L}{\partial \rho} = \sum_{i=1}^{n} \frac{q_{i1}\, q_{i2}\, \phi_2}{\Phi_2}$$

$$g_{i1} = \phi(w_{i1})\, \Phi\left[\frac{w_{i2} - \rho_i^* w_{i1}}{(1-\rho_i^{*2})^{1/2}}\right]$$

$$g_{i2} = \phi(w_{i2})\, \Phi\left[\frac{w_{i1} - \rho_i^* w_{i2}}{(1-\rho_i^{*2})^{1/2}}\right]$$

Example:

Hospital $_{it}$ = 1 if HospVis > 0, 0 oth.

DocVis $_{it}$ = 1 if DocVis > 0, 0 oth.

let $\underset{\sim}{x}_1 = \underset{\sim}{x}_2$ = Const, Female, Age, Inc, Kids, Educ, Married.

Gender Econ Courses of Liberal Arts Colleges

$$G = \begin{cases} 1 & \text{if there is a gender econ course} \\ 0 & \text{else} \end{cases}$$

$$W = \begin{cases} 1 & \text{women's studies' program} \\ 0 & \text{else} \end{cases}$$

$$\text{PROB}[G=1, W=1 \mid x_G, x_W]$$
$$= \Phi_2(x_G^T \beta_G + \gamma W, x_W^T \beta_W, \rho)$$

x_G & x_W are sets of indep variables.

Burnett 1997

132 schools.
58 women's studies programs
31 Gender Econ courses.

Academic Reputation appears in Both x_G & x_W. To find its effect on the PROB of Gender Econ Take some effort.

Using Conditional Prob.

$$E[G \mid x_G, x_W] = \text{Prob}[W=1]\, E[G \mid W=1, x_G, x_W]$$
$$+ \text{Prob}[W=0]\, E[G \mid W=0, x_G, x_W]$$

$$= \Phi_2(x_G^T\beta_G + \gamma,\ x_W^T\beta_W, \rho)$$
$$+ \Phi_2(x_G^T\beta_G,\ -x_W^T\beta_W, -\rho)$$

Take the derivatives of this w/respect to each.

If the variable is Binary (e.g., W) then the derivatives may not give such a good approx.
Use Finite difference

$$\text{Prob}[G=1 \mid W=1] - \text{Prob}[G=1 \mid W=0]$$