IIA

A disadvantage of MNL and CL is that the odds ratios are independent of the other alternatives

TAKE 3 ALTERNATIVES:

~~Comm~~ Commute = { Alone (A), Bus (B), Carpool (C).

$$\text{PROB}(A) = \frac{e^{x\beta_A}}{1 + e^{x\beta_A} + e^{x\beta_B}}$$

$$\text{PROB}(B) = \frac{e^{x\beta_B}}{1 + e^{x\beta_A} + e^{x\beta_B}}$$

$$\text{PROB}(C) = \frac{1}{1 + e^{x\beta_A} + e^{x\beta_B}}$$

Hence, $\frac{\text{PROB}(A)}{\text{PROB}(C)} = e^{x\beta_A}$ $\frac{\text{PROB}(B)}{\text{PROB}(C)} = e^{x\beta_B}$

Odds ratios do not depend on other Alternatives. ~~Prob~~ Relative Prob of commuting alone is not affected by Bus (i.e. β_B)

Let's say

$$P(A) = 1/2$$

$$P(B) = 1/3$$

$$P(C) = 1/6$$

Odds Ratios.

$$\frac{P(A)}{P(C)} = 3 \qquad \frac{P(B)}{P(C)} = 2$$

Now, suppose we add an irrelevant alternative.

Red Bus - RB
Blue Bus - BB

Assuming people are indifferent between the Blue & Red colors of bus, we'd expect this to have no effect on the prob of going alone or of carpooling. –

Unfortunately, though, in the MNL and CL it does.

– How so? Well, the estimate of β_B depends on the characteristics of the individual (MNL) or attributes (relative which are) they should not be affected by the color of the bus.

So: $\beta_{RB} = \beta_{BB} = \beta_B$

$$\text{Prob}(A) = \frac{e^{x\beta_A}}{1 + e^{x\beta_{RB}} + e^{x\beta_{BB}} + e^{x\beta_A}}$$

$\beta_{RB} = \beta_{BB} = \beta_A$

$$= \frac{e^{x\beta_A}}{1 + 2e^{x\beta_B} + e^{x\beta_A}}$$

which is different than before.

also, the odds ratios don't change

$$\frac{\text{Prob}(A)}{\text{Prob}(C)} = e^{x\beta_A}$$

$$\frac{\text{Prob}(BB)}{\text{Prob}(C)} = \frac{\text{Prob}(RB)}{\text{Prob}(C)} = e^{x\beta_B}$$

MNL	Common Sense
$P(A) = 3/8$	$P(A) = 1/2$
$P(RB) = 1/4$	$P(RB) = P(BB) = 1/6$
$P(BB) = 1/4$	$P(C) = 1/6$
$P(C) = 1/8$	

When IIA fails, an alternative Estimator is needed.

(1) Multivariate Probit

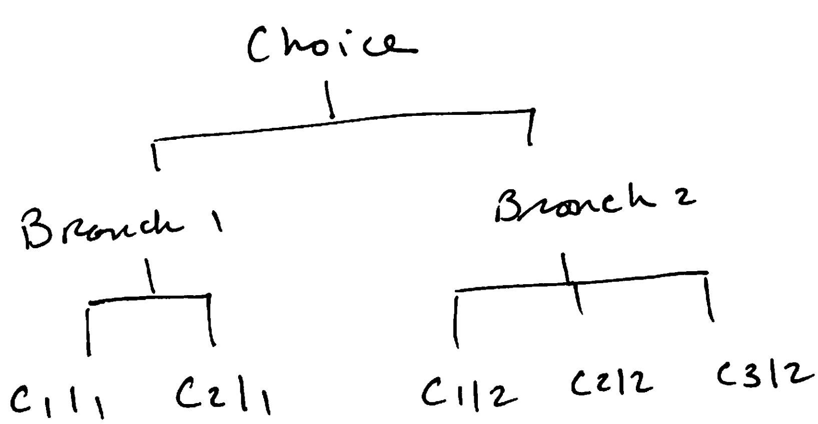
$$U_{ij} = x_{ij}^{T} \beta + \varepsilon_{ij}$$

$$j = 1, 2, \ldots, J$$

$$\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{iJ} \sim N(0, \Sigma)$$

In small dimensional problems (J small), this is computationally feasible.

(2) Nested Logit



"twigs"

Test for IIA

Hausman & McFadden (1984)

If an alternative really is irrelevant, then omitting it will not change the parameter estimates very much, if any. Excluding an irrelevant choice won't be efficient But it is consistent.

If the odds ratios do depend on the alternatives, then the parameter estimates obtained when these choices are excluded are inconsistent

Let $\hat{\beta}_s$, $\hat{V}_s$ be the estimates of β_s with subset of the choices. and est. covariance

Let $\hat{\beta}_f$, $\hat{V}_f$ be the estimates

of β and $\mathrm{Cov}(\hat{\beta}_f)$ using the full

set

$$\chi^2_k = \underset{1 \times k}{(\hat{\beta}_s - \hat{\beta}_f)^T} \underset{k \times k}{[\hat{V}_s - \hat{V}_f]^{-1}} \underset{k \times 1}{(\hat{\beta}_s - \hat{\beta}_f)}$$

$\hat{\beta}_s$, $\hat{\beta}_f$ are $k \times 1$

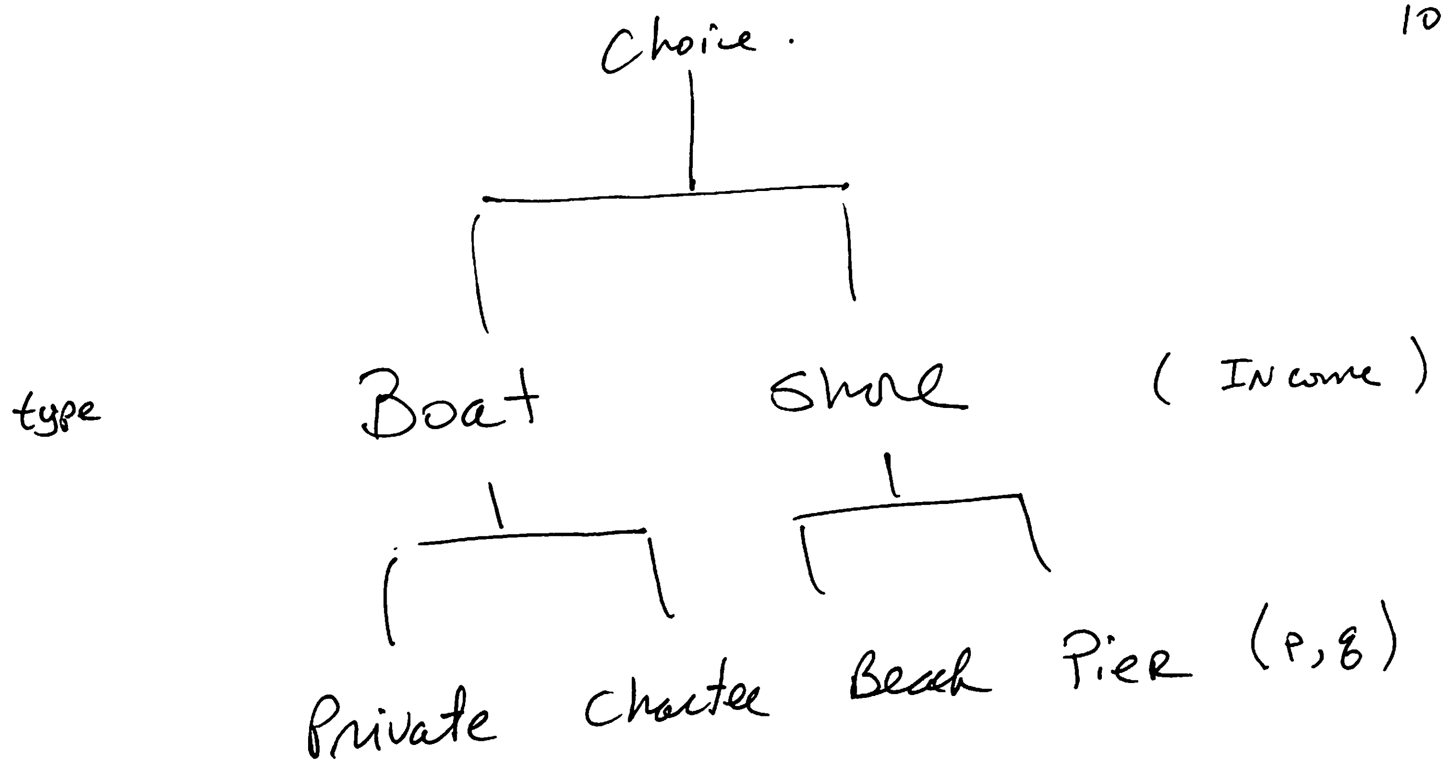
χ^2_k if H_0: True.

H_0: one (or more) choices irrelevant

H_A: not irrelevant.

$$\ln L = \sum_{i=1}^{n} \ln \left[\text{PROB}(\text{twig} \mid \text{branch})_i \times \text{PROB}(\text{branch})_i \right]$$

Recall $\text{PROB}(A, B) = \text{PROB}(A \mid B)\,\text{PROB}(B)$

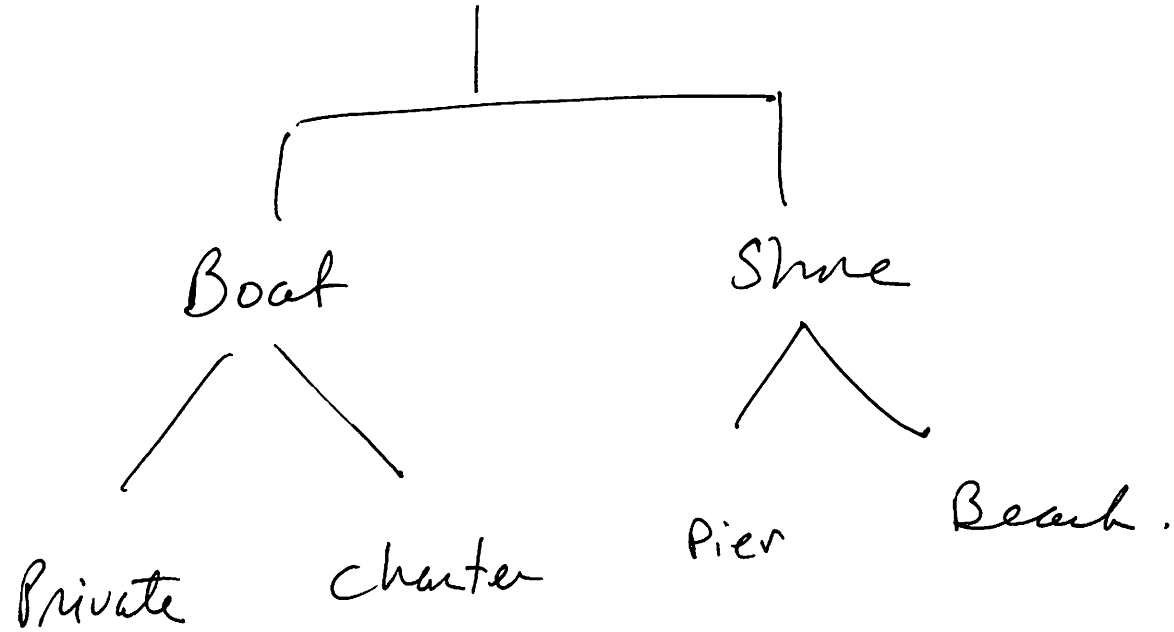


IIA applies within each Branch
But not across Branches.

nlogitgen type = fishmode (shore: pier | beach,
boat: private | charter)

nlogittree fishmode type, choice(d)

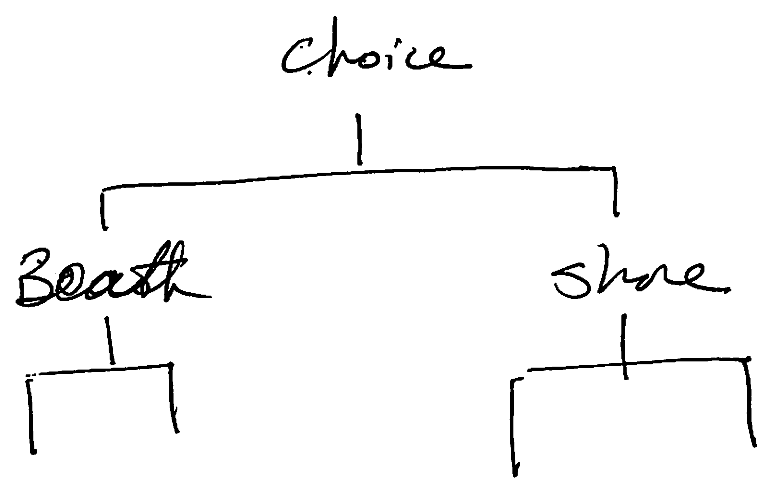
```
.
. * estimate the model
. nlogit d p q || type:, base(shore) || fishmode: income, case(id) notree nolog

RUM-consistent nested logit regression          Number of obs      =      4728
Case variable: id                               Number of cases    =      1182

Alternative variable: fishmode                  Alts per case: min =         4
                                                               avg =       4.0
                                                               max =         4

                                                   Wald chi2(5)    =    212.37
Log likelihood = -1192.4236                        Prob > chi2     =    0.0000

------------------------------------------------------------------------------
           d |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
fishmode     |
           p |  -.0267625   .0018937   -14.13   0.000    -.0304741    -.023051
           q |   1.340078   .3080391     4.35   0.000     .7363329    1.943824
------------------------------------------------------------------------------
fishmode equations
-------------+----------------------------------------------------------------
beach        |
      income |     (base)
       _cons |     (base)
-------------+----------------------------------------------------------------
charter      |
      income |  -8.402969   78.34197    -0.11   0.915    -161.9504    145.1445
       _cons |   69.96747   558.6645     0.13   0.900    -1024.995     1164.93
-------------+----------------------------------------------------------------
pier         |
      income |  -9.458769   80.28494    -0.12   0.906    -166.8144    147.8968
       _cons |   58.94565    500.589     0.12   0.906    -922.1908    1040.082
-------------+----------------------------------------------------------------
private      |
      income |  -1.634704   8.583156    -0.19   0.849    -18.45738    15.18797
       _cons |   37.51814   230.7252     0.16   0.871     -414.695    489.7313
------------------------------------------------------------------------------
dissimilarity parameters
-------------+----------------------------------------------------------------
type         |
  /shore_tau |   83.46671   718.2333                     -1324.245    1491.178
   /boat_tau |   52.56573   542.7797                     -1011.263    1116.394
------------------------------------------------------------------------------
LR test for IIA (tau = 1):             chi2(2) =    45.43   Prob > chi2 = 0.0000
------------------------------------------------------------------------------

.
. predict plevel1 plevel2, pr

. tabulate fishmode, summarize(plevel2)

            |     Summary of Pr(fishmode
            |          alternatives)
   fishmode |        Mean   Std. Dev.       Freq.
------------+------------------------------------
      beach |    .1132351   .13335995        1182
    charter |   .38070838   .15724242        1182
       pier |   .15072736   .16982071        1182
    private |   .35532917   .16444521        1182
------------+------------------------------------
      Total |         .25   .19690069        4728
```



in Nested Model, The errors within each Branch are correlated.

ε_{ij}: Private & charter ε_{ij}: Pier & Beach.

CL is a special case where <u>all</u> errors are independent.


choice
Beach
shore

```
use "C:\DATA\stata\mus\mus15data.dta", clear

log using "H:\Documents and Settings\Lee\My Documents\Document\stata\asclogit.smcl",
replace

generate id = _n
list mode price pbeach  ppier pprivate pcharter in 1/4, clean

* Multinomial Logit -- case specific vars only in short form
mlogit mode income, baseoutcome(1)

* Convert data to long form for asclogit
reshape long d p q, i(id) j(fishmode beach pier private charter) string
list in 1/4

* Drop the variables that don't make sense in long form
drop mode price crate

* Conditional logit with case and alternative specific variables
asclogit d p q, case(id) alternatives(fishmode) casevars(income)

* Predicted probabilities of choice of each mode and compare to actual freqs
predict pasclogit, pr
table fishmode, contents(mean d mean pasclogit sd pasclogit) cellwidth(15)

* Marginal effect at mean of change in price
estat mfx, varlist(p)

* MNL is CL with no alternative specific regressors
asclogit d, case(id) alternatives(fishmode) casevars(income) basealternative(beach)

* Nested Logit
* define the tree
nlogitgen type = fishmode(shore: pier | beach, boat: private | charter)

* check the tree
nlogittree fishmode type, choice(d)

* estimate the model
nlogit d p q || type:, base(shore) || fishmode: income, case(id) notree nolog

predict plevel1 plevel2, pr
tabulate fishmode, summarize(plevel2)
```