

Multinomial Probit

$$u_{ij} = \gamma_{ij}^T \beta + z_i^T \delta_j + \varepsilon_{ij}$$

$\varepsilon \sim N(0, \Sigma)$ where

$$\varepsilon = (\varepsilon_{i1}, \varepsilon_{i2}, \dots, \varepsilon_{im})$$

$$\text{Prob}(y_i = j) = \text{Prob}(\varepsilon_{ik} - \varepsilon_{ij} \leq (\gamma_{ij} - \gamma_{ik})^T \beta + z_i^T (\delta_j - \delta_k) \text{ for all } k.$$

(m-1) dimensional integral for which there is no closed form solution.

When $\Sigma = I$, then the computations are reasonably simple - STATA mprobit.

otherwise, you'll use simulated ML

Take a sample of N independent
OBS. The MLE maximizes

$$\ln L(\underline{\theta}) = \sum_{i=1}^N \sum_{j=1}^m y_{ij} \ln F_j(\gamma_i, \underline{\theta})$$

$$\hat{\underline{\theta}} \approx N(\underline{\theta}, -E \left\{ \frac{\partial^2 \ln L}{\partial \underline{\theta} \partial \underline{\theta}^T} \right\}^{-1})$$

with

$$\underline{\theta} = \begin{pmatrix} \beta \\ \delta_1 \\ \vdots \\ \delta_m \end{pmatrix}$$

The log-likelihood is approximated by
a simulator $\tilde{F}_j(\gamma_i, \underline{\theta})$ which is
based on S draws.

ex. given current estimate $\hat{\underline{\theta}}$, take
 S draws of $\epsilon_i \sim N(0, \hat{\Sigma})$ and let
 $\tilde{F}_j(\gamma_i, \hat{\underline{\theta}})$ be the proportion of

$$S \text{ for which } \epsilon_{ik} = \epsilon_i \cdot \frac{(\gamma_{ij} - \gamma_{in})^T \hat{\beta}}{\hat{\sigma}_k} + z_i^T (\hat{\gamma}_j - \hat{\gamma}_k)$$

More complicated simulations have
been proposed that work better
for \tilde{F} when \tilde{F} is small.

(GHK - Geweke-Hajivassiliou-Keowne)

- STATA

asmp.robust - some basic
syntax or asclost
and require data in long form.

Another Solution to IIA

Random Parameters Logit

$$\begin{aligned} \mu_{ij} &= \gamma_{ij}^T \beta_i + z_i^T \gamma_{ji} + \epsilon_{ij} \\ &= \gamma_{ij}^T \beta + z_i^T \gamma_j + \underbrace{\gamma_{ij}^T v_i + z_i^T w_{ji}}_{\text{combined error}} + \epsilon_{ij} \end{aligned}$$

$$\beta_i = \beta + v_i \quad v_i \sim N(0, \Sigma_B)$$

$$\gamma_{ij} = \gamma_j + w_{ij} \quad w_{ij} \sim N(0, \Sigma_{\gamma_j})$$

The combined ~~of~~ error term is now correlated across alter.

Revelt & Train (1998)

Again, the simulation method is used to estimate the parameters.

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name: <unnamed>
log: c:\temp\stata\one.smcl
log type: smcl
opened on: 5 Dec 2011, 12:02:50

```

- ```

1 . generate id = _n
2 . list mode price pbeach ppier pprivate pcharter in 1/4, clean

```

|    | mode    | price  | pbeach  | ppier   | pprivate | pcharter |
|----|---------|--------|---------|---------|----------|----------|
| 1. | charter | 182.93 | 157.93  | 157.93  | 157.93   | 182.93   |
| 2. | charter | 34.534 | 15.114  | 15.114  | 10.534   | 34.534   |
| 3. | private | 24.334 | 161.874 | 161.874 | 24.334   | 59.334   |
| 4. | pier    | 15.134 | 15.134  | 15.134  | 55.93    | 84.93    |

- ```

3 .
4 . * Convert data to long form for asclogit
5 . reshape long d p q, i(id) j(fishmode beach pier private charter) string

```

```

Data                                wide  ->  long
-----
Number of obs.                      1182 ->  4728
Number of variables                   17  ->   9
j variable (4 values)                 ->  fishmode
xij variables:
    dbeach dpier ... dcharter        ->  d
    pbeach ppier ... pcharter        ->  p
    qbeach qpier ... qcharter        ->  q

```

- ```

6 . list in 1/4

```

|    | id | fishmode | mode    | price  | crate | d | p      | q     | income   |
|----|----|----------|---------|--------|-------|---|--------|-------|----------|
| 1. | 1  | beach    | charter | 182.93 | .5391 | 0 | 157.93 | .0678 | 7.083332 |
| 2. | 1  | charter  | charter | 182.93 | .5391 | 1 | 182.93 | .5391 | 7.083332 |
| 3. | 1  | pier     | charter | 182.93 | .5391 | 0 | 157.93 | .0503 | 7.083332 |
| 4. | 1  | private  | charter | 182.93 | .5391 | 0 | 157.93 | .2601 | 7.083332 |

- ```

7 .
8 . * Drop the variables that don't make sense in long form
9 . drop if fishmode == "charter" | mode == 4
   (2538 observations deleted)
10 . asmprobit d p q, case(id) alternatives(fishmode) casevars(income) correlation(unstructured) structu
    > ral vce(robust)
note: variable p has 106 cases that are not alternative-specific: there is no within-case
    variability

```

```

Iteration 0:  log simulated-pseudolikelihood = -493.8207
Iteration 1:  log simulated-pseudolikelihood = -483.41654 (backed up)
Iteration 2:  log simulated-pseudolikelihood = -482.98783 (backed up)
Iteration 3:  log simulated-pseudolikelihood = -482.9415 (backed up)
Iteration 4:  log simulated-pseudolikelihood = -482.67112
Iteration 5:  log simulated-pseudolikelihood = -482.51402
Iteration 6:  log simulated-pseudolikelihood = -482.44493
Iteration 7:  log simulated-pseudolikelihood = -482.39599
Iteration 8:  log simulated-pseudolikelihood = -482.37574
Iteration 9:  log simulated-pseudolikelihood = -482.35251
Iteration 10: log simulated-pseudolikelihood = -482.30752
Iteration 11: log simulated-pseudolikelihood = -482.30473
Iteration 12: log simulated-pseudolikelihood = -482.30184
Iteration 13: log simulated-pseudolikelihood = -482.30137
Iteration 14: log simulated-pseudolikelihood = -482.30128
Iteration 15: log simulated-pseudolikelihood = -482.30128

```


13 . estat mfx

Pr(choice = beach) = .14878681

variable	dp/dx	Std. Err.	z	P> z	[95% C.I.]	X
P						
beach	-.003556	.001052	-3.38	0.001	-.005619 -.001494	92.756
pier	.001307	.000889	1.47	0.141	-.000435 .00305	92.756
private	.002249	.000346	6.51	0.000	.001572 .002926	61.877
Q						
beach	.213103	.06986	3.05	0.002	.07618 .350025	.23426
pier	-.078342	.044395	-1.76	0.078	-.165356 .008671	.16389
private	-.13476	.049896	-2.70	0.007	-.232555 -.036965	.16753
casevars						
income	.001499	.006416	0.23	0.815	-.011075 .014074	4.2346

Pr(choice = pier) = .16760542

variable	dp/dx	Std. Err.	z	P> z	[95% C.I.]	X
P						
beach	.001307	.000889	1.47	0.141	-.000434 .003049	92.756
pier	-.004163	.000953	-4.37	0.000	-.00603 -.002295	92.756
private	.002855	.000381	7.49	0.000	.002108 .003602	61.877
Q						
beach	-.078342	.044388	-1.76	0.078	-.165341 .008657	.23426
pier	.24944	.081484	3.06	0.002	.089733 .409146	.16389
private	-.171098	.068782	-2.49	0.013	-.305909 -.036287	.16753
casevars						
income	-.022517	.007227	-3.12	0.002	-.036682 -.008352	4.2346

Pr(choice = private) = .68359653

variable	dp/dx	Std. Err.	z	P> z	[95% C.I.]	X
P						
beach	.002249	.000346	6.51	0.000	.001572 .002926	92.756
pier	.002856	.000381	7.49	0.000	.002108 .003603	92.756
private	-.005105	.000643	-7.94	0.000	-.006364 -.003845	61.877
Q						
beach	-.134767	.049895	-2.70	0.007	-.232559 -.036975	.23426
pier	-.171109	.068784	-2.49	0.013	-.305923 -.036295	.16389
private	.305877	.116982	2.61	0.009	.076596 .535158	.16753
casevars						
income	.021019	.009799	2.14	0.032	.001813 .040225	4.2346

14 . log close

name: <unnamed>
 log: c:\temp\stata\one.smcl
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