Fixed Effects Model

$$\underset{\sim}{y} = X_s \underset{\sim}{\beta} + D \underset{\sim}{\alpha} + \underset{\sim}{\varepsilon}$$

$$D = I_N \otimes \underset{\sim}{i}_T$$

FE estimator:

use M_D to remove dummy vars from model.

$$M_D = I - P_D \qquad P_D = D(D^T D)^{-1} D^T$$

$$M_D \underset{\sim}{y} = M_D X_s \beta + M_D D \underset{\sim}{\alpha} + M_D \underset{\sim}{\varepsilon}$$
$$= M_D X_s \beta + M_D \underset{\sim}{\varepsilon}$$

$$\hat{\beta}_{FE} = (X_s^T M_D X_s)^{-1} X_s^T M_D \underset{\sim}{y}$$

Between Effects Model

$$\bar{y}_{i.} = \bar{x}_{i.}^T \beta + \alpha + \bar{e}_{i.} \qquad i = 1, \cdots, n$$

This Model can be obtained using $P_D = D(D^T D)^{-1} D^T$

Recall:

$$P_D = (I_N \otimes \underset{\sim}{i}_.)\left[(I_N \otimes \underset{\sim}{i}_.)^T (I_N \otimes \underset{\sim}{i}_.)\right]^{-1} (I_N \otimes \underset{\sim}{i})^T$$

$$= I_N \otimes \underset{\sim}{i}_T \underset{\sim}{i}_T^T / T$$

Model

$$P_D \underset{\sim}{y} = P_D X \underset{\sim}{\beta} + P_D D \underset{\sim}{\alpha} + P_D \underset{\sim}{\varepsilon}$$

$$P_D D = D$$

But $\underset{\sim}{\alpha}$ are not separately identified

Notes:

Asymptotics

n - # individuals or groups

T = time Periods

nT - Total sample size
(balanced)

1) Suppose n is fixed. As $T \to \infty$
The number ~~for~~ of fixed effects
Remains constant. So, we
have more obs to estimate
both slopes and α_i as $T \to \infty$
$\Rightarrow$ Standard Asympt Theory
Applies

2) Suppose $\underline{n}$ is $\underline{\to \infty}$ with $\underline{T}$ fixed

$\Rightarrow$ # parameters to estimate
is also converging to ∞
Then $\hat{\alpha}_i$ are NOT
consistent in fixed Effects
Estimates.

3) Even if $n \to \infty$ and T- fixed

$\hat{\beta}_{FE}$ is consistent for β

(miraculously!)

4) An estimate of σ_α^2 is

the sample variance in $\hat{\alpha}_i$

Obviously, the larger the number of $\hat{\alpha}_i$ the better (finite) (except if $n \to \infty$.)

Cluster Std. Errors

Consider the Pooled Model

$$y_{it} = x_{it}^T \beta + \alpha + \varepsilon_{it}$$

if $\alpha_i \neq \alpha$ for all i

then LS is inconsistent.

for fixed effects Models. If α_i correlated with x_{si}

If α_i are Random, LS might be consistent. Basically there is autocorrelation in the model $Cov(\alpha_{it}, \alpha_{js}) = \sigma_\alpha^2$ $\begin{matrix} t \neq s \\ i = j \end{matrix}$

And the usual OLS Result Applies.

Look at the i^{th} Group or ~~the~~ individual

$$\underset{\sim\, T\times 1}{y_i} = \overset{X_i \beta}{\cancel{}} + \underset{\sim}{\eta_i}_{T\times 1}$$

$$\eta_{it} = \alpha_i + \varepsilon_{it}$$

$\underset{\sim}{\beta}$ includes the constant

X_i $\underset{\sim}{}_{T\times K}$ matrix and include $\underset{\sim}{i}_T$

$$X_i = \left[\, \underset{\sim}{i}_T \;\vdots\; X_{si} \,\right]$$

$$Cov(\eta_i) = \Sigma = \begin{bmatrix} \sigma_\varepsilon^2 + \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \ddots & & \vdots \\ \vdots & & \ddots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 & \sigma_\varepsilon^2 + \sigma_\alpha^2 \end{bmatrix}$$

So, there is intragroup autocorrelation

1 Estimate model using LS
(looks like pooled model)

Get LS estimates of Residuals
for each group

$\hat{n}_i$

Est "Clustered" Robust Cov
matrix is

$$\frac{1}{n}\left[\frac{1}{n}\sum_{i=1}^{n} X_i^T X_i\right]^{-1}\left[\frac{1}{n}\sum_{i=1}^{n} X_i^T \hat{n}_i \hat{n}_i^T X_i\right]\left[\frac{1}{n}\sum_{i=1}^{n} X_i^T X_i\right]^{-1}$$

SANDWICH Cov

use these with

Pooled LS estimates.

$$i_{nT} = i_n \otimes i_T$$

$$P_D = I_N \otimes i_T i_T^T / T$$

$$\therefore$$

$$P_D \, i_{nT} = \left(I_N \otimes \frac{i_T i_T^T}{T} \right) \left(i_n \otimes i_T \right)$$

$$= i_n \otimes \frac{i_T i_T^T i_T}{T}$$

$$= i_n \otimes i_T \, T/T = i_n \otimes i$$

$$= i_{nT} \; !$$

$$y = X\beta + \underset{\sim}{u} \quad u \sim (0, \Omega)$$

use LS with A "Robust"
covariance estimator.

$$\Omega = \begin{bmatrix} \sigma_1^2 & & & 0 \\ & \sigma_2^2 & & \\ & & \ddots & \\ 0 & & & \sigma_T^2 \end{bmatrix}$$

Heteroscedastic.

(1) use LS with
White's cov estimator

(2) use GLS (or FGLS)
with FGLS cov.

$$y_{it} = x_{it}^T \underset{\sim}{\beta} + \alpha + \varepsilon_{it}$$

(1) use LS (Pooled)
and Cluster Std errors

(2) GLS or FGLS in The
Random effects estimator.
That specifically models
The intragroup Autocorrelations
σ_α^2.

HAUSMAN TEST

Fixed Effects vs Random Effects

$\downarrow$ if X_i & α_i Correlated

$\downarrow$ only if α_i & X_i uncorrelated

H_0: $X_i \alpha_i$ uncorrelated

H_A: $X_i \alpha_i$ Correlated.

Principle is to compare two estimators

(1) Efficient under H_0: (and both consistent)

(2) Not efficient but consistent under H_A:

→ Both are consistent under H_0 and should produce estimates that are similar — at the same time (1) will be more efficient than (2)

Are Effects Random?

$\alpha_i \sim (0, \sigma_\alpha^2) \Rightarrow \alpha_i$ Random

If $\sigma_\alpha^2 = 0$ Then They are $\underline{\underline{\text{NOT}}}$ Random.

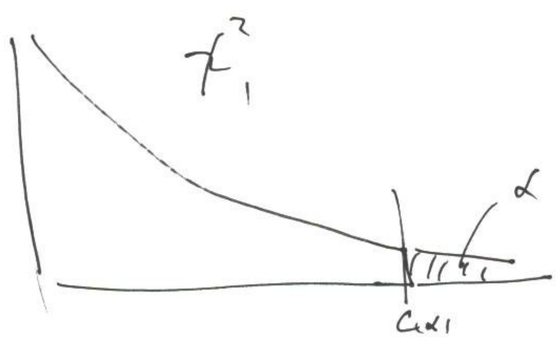
$H_0: \sigma_\alpha^2 = 0$

$H_A \quad \sigma_\alpha^2 > 0$

$$\lambda = \frac{NT}{2(T-1)}\left[\frac{\bar{\underset{\sim}{e}}^T (I_N \otimes \underset{\sim}{i}_T \underset{\sim}{i}_T^T)\bar{\underset{\sim}{e}}}{\bar{\underset{\sim}{e}}^T\bar{\underset{\sim}{e}}} - 1\right]^2 \xrightarrow{d} \chi_1^2$$

if Ho True.

$\bar{\underset{\sim}{e}}$ residuals from $\underline{\underline{\text{Pooled}}}$ Regression



Reject if $\lambda > c(\alpha)$ (Effects Random)

Prediction (RE)

(1) $E(y_{it} \mid x_{it})$

$$\hat{\alpha} + \sum_{k=2}^{K} \hat{\beta}_k x_{kit}$$

Average over all time periods & Groups given certain characteristics (No ind. Effects)

(2) Predict y for i^{th} individual $\mid x_{it}$

$$\hat{\alpha} + \sum_{k=2}^{K} \hat{\beta}_k x_{kit} + \underline{\alpha}_i$$

$$\hat{\alpha}_i = \frac{\hat{\sigma}_\alpha^2}{\hat{\sigma}_1^2} \underset{\sim}{i}_T^T (\underset{\sim}{y}_i - \underset{\sim}{x}_i^T \hat{\beta}_{RE})$$

$$= \frac{\sigma_\alpha^2}{\sigma_1^2/T} \underset{\sim}{i}_T^T \frac{(y_i - x_i^T \hat{\beta}_{RE})}{T}$$

$$= \frac{\sigma_\alpha^2}{\sigma_1^2/T} \hat{\bar{e}}_{i\cdot}$$

verage residual for

i^{th} GROUP

weighted By Proport
of variance explaine
By individual, Relat
to Variance of Beta
Estimator.